Abstract

Subspace-based algorithms for system identification have lately been suggested as alternatives to more traditional techniques. Variants of the MOESP type of subspace algorithms are in addition to open-loop identification applicable to closed-loop and errors-in-variables identification. In this paper, a new instrumental variable approach to subspace identification is presented. It is shown how existing MOESP-algorithms can be derived within the proposed framework, simply by changing instruments and weighting matrices. A noteworthy outcome of the analysis is that an improvement of an existing MOESP method for errors-in-variables identification can be proposed.

Keywords: Identification algorithms; System identification; Subspace methods; Instrumental variables

1. Introduction

Subspace-based methods for State-Space System Identification (4SID) methods are emerging identification techniques (recent overviews can be found in e.g. Van Overschee & De Moor, 1996; Viberg, 1995). The 4SID methods are attractive since a state-space realization is estimated directly from input-output data, without non-linear optimization as generally required by the Prediction Error Method (PEM) (cf. Ljung, 1987; Söderström & Stoica, 1989). Since 4SID methods typically are implemented using computational tools such as the QR-factorization and the singular value decomposition (SVD), they are also numerically reliable.

Examples of popular 4SID methods are PO/PI-MOESP (Verhaegen, 1994), and N4SID (Van Overschee & De Moor, 1996). N4SID and PO/PI-MOESP are in principle applicable only to open-loop identification. In Chou and Verhaegen (1997), however, a MOESP-variant was proposed which is applicable also to closed-loop and errors-in-variables (EIV) identification, and in Van Overschee and De Moor (1997) an N4SID-like method for identification of closed loop systems was described.

In this paper, an Instrumental Variable (IV) approach to subspace identification is presented. The main motivation of our work has been to provide means for analyzing existing methods, rather than to derive new algorithms. Although of great importance when writing numerically reliable identification software, the intensive application of linear algebra tools in 4SID tends to complicate the relationship with traditional identification methods. For example, 4SID methods are commonly derived from quantities obtained from a QR-factorization, whereas traditional identification methods are derived using auto- and cross-correlations of the input/output data.

IV-interpretations of 4SID algorithms have previously been studied in the IV-4SID framework of Viberg (1995). The present paper investigates an alternative IV-approach to subspace identification. Based on the new interpretations, an important contribution of the paper is that an improvement of the EIV-algorithm of Chou and Verhaegen (1997) is proposed.
2. Problem formulation

Suppose that the plant depicted in Fig. 1 can be described by the following discrete-time \( n \)th order time-invariant state-space model with \( p \) undisturbed outputs collected in \( \tilde{y}(t) \), and \( m \) undisturbed inputs collected in \( \tilde{u}(t) \):

\[
x(t + 1) = Ax(t) + B\tilde{u}(t) + w(t),
\]

\[
y(t) = Cx(t) + D\tilde{u}(t).
\]  

(1)

Here, \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), \( D \in \mathbb{R}^{p \times m} \), and \( x(t) \) is the \( n \)-dimensional state vector. The measured input and output signals \( y(t) \) and \( u(t) \) are modeled as

\[
u(t) = \tilde{u}(t) + f(t),
\]

\[
y(t) = \tilde{y}(t) + t(t).
\]  

(2)

(3)

The problem of interest is to estimate a state space realization \( \{A, B, C, D\} \), given measurements of \( \tilde{u}(t) \) and \( \tilde{y}(t) \). The system order \( n \) typically needs to be estimated as well. The fact that we account for the possibility that the input signal is not exactly known, makes the problem difficult, and is often referred to as an errors-in-variables (EIV) problem. For a discussion on the applicability of EIV models, see e.g. Cedervall and Stoica (1996), Schoukens and Pintelon (1991). Since the main scope of the present paper is on interpretations and extensions of EIV models, see e.g. Cedervall and Stoica (1996), (EIV) problem. For a discussion on the applicability of EIV models, see e.g. Cedervall and Stoica (1996), since the main scope of the present paper is on interpretations and extensions of 4SID in general, a more complete discussion of the EIV problem is omitted.

The following assumptions are assumed to hold throughout the paper:

A1: The system (1) is asymptotically stable.

A2: The noise sequences \( \{w(t)\}, \{e(t)\} \) and \( \{f(t)\} \) all consist of zero-mean and independent random variables. The disturbances affecting the plant are the process noise \( w(t) \), and the measurement disturbances \( \epsilon(t), \delta(t) \), where

\[
E, \left[ \begin{array}{c|c|c}
w(t) & v(t) & f(t)
\end{array} \right]^T = \left[ \begin{array}{ccc}
R_{ww} & R_{we} & R_{wf}
R_{ew} & R_{ee} & R_{ef}
R_{fw} & R_{ef} & R_{ff}
\end{array} \right] \delta, \epsilon, \delta, \epsilon.
\]  

(4)

The following input–output relation is then easily derived (see Chou & Verhaegen, 1997):

\[
y(t) = \Gamma_x \epsilon(t) + \Phi_x u(t) - \Phi_y \delta(t) + \Psi_x w(t) + \epsilon(t).
\]  

(8)

The extended observability matrix \( \Gamma_x \) is defined as

\[
\Gamma_x = [C^{T}, (CA)^{T}, \ldots, (CA^{n-1})^{T}]^{T}.
\]  

(9)

Matrices \( \Phi_x \) and \( \Psi_x \) are lower (block) triangular and Toeplitz. The key step in most subspace methods is to build a (block) Hankel matrix \( Y_{f} \) as

\[
Y_{f} = [y_{f}(\beta + 1), \ldots, y_{f}(\beta + N)]
\]

\[
= \Gamma_x \Gamma_x + \Phi_x U_{t} - \Phi_y F_{t} + \Psi_x W_{t} + V_{t},
\]  

(10)

where the matrices \( X_{f}, U_{t}, F_{t}, W_{t} \) and \( V_{t} \) are constructed conformably with \( Y_{f} \). Here it is assumed that measurements of \( y(t) \) and \( u(t) \) are available for \( t = 1, \ldots, N + \alpha + \beta - 1 \).
3.2. The SIV approach

In this section, the Subspace-based Identification using Instrumental Variables (SIV) approach to identification is presented. Suppose that an \( n_x \times N \) dimensional IV-matrix
\[
\Xi = [\xi(\beta + 1), \ldots, \xi(\beta + N)]
\]
is available, and that \( \Xi \) is constructed such that
\[
\lim_{N \to \infty} [W_f \quad V_f \quad F_f] \Xi^T = 0
\]
where the above limits are assumed to exist with probability one (w.p.1). Following the steps of the “typical” IV approach as described in (Söderström & Stoica, 1989, Chapter 8), Eq. (10) is correlated with the instruments:
\[
\tilde{R}_{\gamma} \Xi = \Gamma_x R_{\gamma} x + \Phi_x R_{\gamma} u + \tilde{R}_{n_x} z
\]
where \( \tilde{R}_{\gamma} \Xi = Y_{\gamma} \Xi^T \). The matrices \( \tilde{R}_{\gamma}, \tilde{R}_{\gamma} x \), and \( \tilde{R}_{n_x} \) are defined conformably with \( \tilde{R}_{\gamma} \). For notational convenience, all noise terms have been collected in \( N_t = \Psi_t W_t + \Psi_t F_t \). As long as the IV-vector \( \xi(t) \) fulfills condition (12),
\[
\tilde{R}_{\gamma} \Xi \rightarrow R_{\gamma} \Xi = \Gamma_x R_{\gamma} x + \Phi_x R_{\gamma} u \quad \text{w.p.1 as } N \rightarrow \infty,
\]
where \( R_{\gamma} \Xi = R_{\gamma} z = E[\gamma_x(t) \xi^T(t)] \), from Eq. (12). It remains to outline a procedure that enables extraction of a state space realization. Following a typical “subspace approach”, we will first concentrate on estimation of \( A_{\gamma} \), where \( A_{\gamma} \) denotes the range space of \( \gamma \).

Before we proceed, let us briefly discuss how the IV-vector \( \xi(t) \) should be chosen. When the system operates in open-loop, and \( f(t) \equiv 0 \), a natural IV-candidate is
\[
\xi(t) = z(t)
\]
where
\[
z(t) = [u(t) \quad y(t) \quad u(t) y(t)]^T.
\]

With this choice of instrument, applying A2 and (Ljung, 1987, Theorem 2.3), the assumed convergence w.p.1 in Eq. (12) holds true. In the EIV-case, a natural choice is
\[
\xi(t) = p(t), \text{ where } p(t) = [u(t) \quad y(t) \quad u(t) y(t)]^T.
\]
Also in this case requirement (12) is true. If \( u(t) \) is included in \( \xi(t) \) in the EIV-case, the influence of the noise will not vanish as \( N \rightarrow \infty \) due to the correlation between \( f(t) \) and the other noise-terms. The choice \( \xi(t) = p(t) \) is applicable also to systems that operate in closed-loop, assuming that the controller is causal, that there is at least a delay in the controller, and that the closed loop system is asymptotically stable, cf. Chou and Verhaegen (1997).

Assume next that the dimension of \( \Xi \) is chosen such that \( n_x \geq n + mx \) (which implicitly sets conditions on \( \beta \)).

The inequality \( n_x \geq n + mx \) implies that \( \dim(A_{\gamma} (R_{\gamma})) \geq n \) where \( \dim(A_{\gamma}(\cdot)) \) denotes the dimensionality of the nullspace of \( \gamma \). In order to enable estimation of \( A_{\gamma} \), compute next
\[
\tilde{R}_{n_x} \tilde{P}_{n_x} = \Gamma_x R_{n_x} \tilde{P}_{n_x} + \tilde{R}_{n_x} \tilde{P}_{n_x}
\]
where \( \tilde{P}_{n_x} \) denotes the orthogonal projection matrix onto the nullspace of \( \tilde{R}_{n_x} \). Assuming that \( \tilde{R}_{n_x} \) has full rank (cf. Section 3.3)
\[
\Pi_{n_x} = \lim_{N \to \infty} \tilde{P}_{n_x} \tilde{P}_{n_x} = I - \tilde{R}_{n_x} (R_{n_x} R_{n_x}^\top)^{-1} R_{n_x} \quad \text{w.p.1}
\]
Since \( \tilde{R}_{n_x} \rightarrow 0 \) w.p.1 as \( N \rightarrow \infty \), and since the projection matrix is bounded:
\[
\lim_{N \to \infty} \tilde{R}_{n_x} \tilde{P}_{n_x} = \Gamma_x R_{n_x} \Pi_{n_x} \quad \text{w.p.1.}
\]
The SIV estimate of the extended observability matrix is based on the SVD
\[
\hat{G} = \hat{R}_{n_x} \hat{P}_{n_x} \hat{W}_R = \hat{Q}_{t} \hat{S}_1 \hat{V}_1^\top + \hat{Q}_2 \hat{S}_2 \hat{V}_2^\top,
\]
where \( \hat{W}_R \) is a weighting matrix. The matrix \( \hat{Q}_{t} \) consists of the \( n \) principal left singular vectors of \( \hat{G}, \hat{S}_1, \text{ and } \hat{S}_2 \) are diagonal matrices with the non-increasing singular values of \( \hat{G} \) on their diagonals. In absence of noise, \( \hat{S}_2 = 0 \). However, in practice \( \hat{S}_2 \neq 0 \), and a decision on the order of the system must be made. The “subspace estimate” is obtained as \( \hat{\Gamma}_x \). Clearly, the consistency of this estimator hinges on the assumption that \( R_{n_x} \Pi_{n_x} \) has rank \( n \). This requirement is discussed in the following section.

3.3. Aspects on consistency

From (18) it is clear that a consistent estimate of \( \hat{\Gamma}_x \) can be obtained if and only if \( \text{Rank}(R_{n_x} \Pi_{n_x}) = n \). The inequality \( n_x \geq n + mx \) is only a necessary requirement for \( \text{Rank}(R_{n_x} \Pi_{n_x}) = n \) to be true. Consider the following result for sufficient conditions:

**Result 1.** If \( f(t) \equiv 0 \), and if \( \xi(t) = z(t) \), consistency of \( \hat{\Gamma}_x \) holds under exactly the same conditions as for IV-4SID, cf. Jansson and Wahlberg (1998).

**Proof.** Note first that
\[
\text{Rank}(R_{n_x} \Pi_{n_x}) = n \Leftrightarrow \text{Rank}(R_{n_x} \Pi_{n_x} R_{n_x}^\top) = n.
\]
Furthermore, since \( R_{n_x} \) has full rank under any of the persistence of excitation conditions stated in Jansson and Wahlberg (1998), \( R_{n_x} R_{n_x}^\top \) is invertible. From Jansson

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1 It is assumed that the involved matrices are appropriately scaled (i.e. with \( \sqrt{N} \)), so that quantities like \( \Xi^T E[\gamma_x(t) \xi^T(t)] = R_{n_x} \), as \( N \rightarrow \infty \).

2 It is possible to include also a row-weighting \( W_L \) in (19). To simplify the presentation, \( W_L \) is however omitted.
and Wahlberg (1998) it then follows that
\[
\text{Rank}(R_{x,t}R_{\hat{u},t}^T, R_{x,t}^TR_{\hat{u},t}) = n
\]
\[
\Rightarrow \text{Rank}
\begin{pmatrix}
R_{x,t}R_{\hat{u},t}^T & R_{x,t}^TR_{\hat{u},t}
\end{pmatrix}
= m + n.
\]
(21)

Finally, since
\[
\begin{pmatrix}
R_{x,t}R_{\hat{u},t}^T & R_{x,t}^TR_{\hat{u},t}
\end{pmatrix}
\begin{pmatrix}
R_{x,t}^T & R_{\hat{u},t}^T
\end{pmatrix}
= \begin{pmatrix}
R_{x,t}R_{\hat{u},t}^T & R_{x,t}^TR_{\hat{u},t}
\end{pmatrix}
, \tag{22}
\]

Result 1 is shown by a direct comparison with the critical rank condition in Jansson and Wahlberg (1998). \square

The corresponding rank conditions for the EIV case (i.e. \( f(t) \neq 0 \) and \( \zeta(t) = p(t) \)) are more complicated, as discussed in Chou and Verhaegen (1997). Unfortunately we have not been able to determine sufficient conditions for guaranteeing that \( R_{x,t}^TR_{\hat{u},t} \) is full rank in this case.

### 3.4. Estimation of system matrices

Estimation of the actual state space realization will not be discussed in any greater detail in the present contribution. The reason is that there exist several well-known approaches to estimate the state space matrices, once the estimated observability matrix \( \hat{P}_s \) is found. The estimates of \( A \) and \( C \) can be found using the shift-invariant structure of \( F_s \), see e.g. Viberg (1995). The \( B \) and \( D \) matrices can for example be obtained as the least squares solution of (linear in \( B \) and \( D \))

\[
\hat{Q}_s^T \hat{R}_{\hat{u},t} \hat{R}_{\hat{u},t}^T \approx \hat{Q}_s^T \Phi_s,
\]
(23)

where \( (-)^\dagger \) denotes the Moore-Penrose pseudo-inverse. The approach in (23) provides consistent estimates of \( B \) and \( D \) if \( \hat{R}_s \) is consistent, and if \( R_{\hat{u},t} \) has full row-rank. In the open-loop scenario \( (\zeta(t) = z(t)) \), \( R_{\hat{u},t} \) has full row-rank if the input \( \hat{u}(t) \) is persistently exciting of order \( x \). The corresponding requirement in the EIV case \( (\zeta(t) = p(t)) \) is more involved. It is for example easy to see that the approach in (23) is not applicable if \( \hat{u}(t) \) is white. A more detailed discussion on how to estimate \( B \) and \( D \) in the EIV scenario can be found in Chou and Verhaegen (1997).

### 4. Relationship with existing subspace methods

The purpose of this section is to analyze the relationship with IV-4SID. In Viberg (1995), IV-4SID algorithms, applicable to the case \( f(t) \equiv 0 \), are “derived” as:

1. Compute \( Y_t^T \hat{P}_{\hat{u},t} \), where \( \hat{P}_{\hat{u},t} \) denotes the orthogonal projection onto \( \mathcal{M}(U_t) \).
2. Correlate with the matrix \( P = [p(\beta + 1), \ldots, \beta + N] \), i.e. compute \( Y_t^T \hat{P}_{\hat{u},t} P^T \).
3. Compute the left singular vectors of \( Y_t^T \hat{P}_{\hat{u},t} P^T W_r \).

From an algebraic point of view, the difference between SIV and IV-4SID is that the correlation and projection steps have been interchanged. This difference may seem minor. However, SIV is applicable to EIV whereas IV-4SID is not, and exactly for this reason!

Although derived in a different manner, there is an interesting relationship between SIV and IV-4SID as the following will demonstrate. Since IV-4SID is not applicable to EIV identification, it is assumed that \( f(t) \equiv 0 \) in the following.

We will restrict the analysis by considering weightings \( W_R \) that are members of the following set:

\[
W_R = (\hat{U}_{u,z}^+ \Sigma \hat{U}_{u,z}^+)^\dagger,
\]
(24)

where \( \Sigma \) is positive definite. It may seem restrictive to only consider weightings as in (24). However, it turns out that this set covers several interesting cases. Here we have implicitly assumed that \( \zeta(t) = z(t) \). With a weighting matrix as in Eq. (24), the estimated observability matrix is obtained from the \( n \) principal eigenvectors of

\[
\hat{G}^T = \hat{R}_{y,z} \hat{U}_{u,z}^+ (\hat{U}_{u,z}^+ \Sigma \hat{U}_{u,z}^+) \hat{U}_{u,z}^+ \hat{R}_{y,z}^T,
\]
(25)

Write the projection matrix \( \hat{P}_{\hat{u},z} \) in a factored manner, \( \hat{P}_{\hat{u},z} = \hat{Q}_s \hat{Q}_s^T \), where \( \hat{Q}_s \hat{Q}_s^T = I \). Since \( \Sigma \) is non-singular and since the columns of \( \hat{Q}_s \) are orthogonal, the set of weightings in (24) can be described as

\[
W_R = \hat{Q}_s (\hat{Q}_s^T \Sigma \hat{Q}_s)^{-1} \hat{Q}_s^T.
\]
(26)

Next we turn to IV-4SID and note that

\[
P \hat{F}_s = [I_{p(m + \beta)} - \hat{P}_{\hat{u},s} \hat{P}_{\hat{u},s}^{-1}] P = \hat{M} Z,
\]
(27)

where \( I_M \) denotes the \( M \)-dimensional identity matrix, and \( Z = [z(\beta + 1), \ldots, z(\beta + N)] \). The matrix \( \hat{M} \) is clearly full rank, and satisfies by construction \( \hat{R}_{u,z} \hat{M}^T = 0 \). Hence, \( \hat{P}(\hat{M}^T) = \hat{P}(\hat{Q}_s^T) \). The matrix \( \hat{Q}_s \) can hence be written as \( \hat{Q}_s \hat{M} = \hat{M} \hat{R} \) for some full rank matrix \( \hat{R} \).

Replacing \( \hat{Q}_s \) with \( \hat{M} \hat{R} \), and using Eq. (26), it follows that

\[
\hat{G}^T = Y_t^T \hat{F}_s \hat{P}(\hat{M} \hat{R} \hat{M}^T)^{-1} P \hat{P}_s \hat{Y}_t^T.
\]
(28)

Phrased differently, suppose that the IV-4SID weighting matrix is denoted \( W_r \). Then the IV-4SID and SIV subspace estimates are identical if \( W_r = (\hat{M} \hat{R} \hat{M}^T)^{-1/2} \). Here, \((-)^{-1/2}\) denotes a symmetric square root of \((-)^{-1}\).

If we further consider the weighted extended IV method (Söderström & Stoica, 1989, Complement C8.5), it is natural to study weightings of the \textit{instruments} \( z(t)/p(t) \) rather than a “matrix weighting” \( W_r \). That is, we will consider IV-vectors \( W_z z(t) \) and \( W_p p(t) \), where \( W_z \) and \( W_p \) are positive definite weighting matrices. An interesting result, is then that the weighting matrix \( W_R \) introduced in Eq. (24) can be interpreted as a pre-whitening of the instruments. That is, if \( \zeta(t) = \Sigma^{-1/2} z(t) \),

\[
\hat{R}_{\hat{y},z} \hat{P}_{\hat{u},z} (\hat{U}_{u,z}^+ \Sigma \hat{U}_{u,z}^+) \hat{U}_{u,z}^+ \hat{R}_{\hat{y},z}^T = \hat{R}_{\hat{y},z} \hat{P}_{\hat{u},z} \hat{R}_{\hat{y},z}^T,
\]
(29)
In other words, in the SIV framework we can forget about \( W_K \), and focus on “pre-whitenings” of the IV-vector \( z(t) \). To prove (29), let the matrix \( A_1 \) be an orthogonal basis for the row-space of \( U_i Z^1 \), and let \( A_2 \) be an orthogonal basis for the orthogonal complement of the row-space of \( U_i Z^1 \). The left hand side of Eq. (29) can now be rewritten as \( \tilde{R}_{n,z} \Sigma^{-1/2} P_{A_1} \Sigma^{-1/2} \tilde{R}_{n,z}^T \). Furthermore, the right hand side of Eq. (29) can be written as \( \tilde{R}_{n,z} \Sigma^{-1/2} \Pi_{A_1} \Sigma^{-1/2} \tilde{R}_{n,z}^T \). Since \( A_1 \Sigma^{-1/2} \) and \( A_2 \Sigma^{1/2} \) are full rank, and since \( A_1 \Sigma^{-1/2}(A_2 \Sigma^{1/2})^T = 0 \), it follows that \( \Pi_{A_1} \Sigma^{-1/2} \) and \( \Pi_{A_2} \Sigma^{1/2} \) projects onto the same space, and Relationship (29) is shown. The calculations for the EIV case are completely analogous. As illustrative examples, consider the following results:

Result 2. Suppose that \( f(t) \equiv 0 \), and let the IV-matrix be defined as a “spatial pre-whitening” of \( Z \):

\[
Z = \tilde{R}_{n,z}^{-1/2} Z. \tag{30}
\]

With \( Z \) defined as in Eq. (30), and with \( W_K = I \), the SIV subspace estimate is identical to the PO-MOESP Verhaegen (1994) subspace estimate. Specifically, if \( U = [U_1^0 U_1^0]^T \), and if \( Z = \tilde{R}_{n,z}^{-1/2} U \), the PI-MOESP subspace estimate is obtained.

Proof. As shown in Viberg (1995), the PO-MOESP subspace estimate is obtained from the \( n \) principal left singular vectors of \( Y_t \Pi_{U_1} P^T (\Pi_{U_1} P)^T \tilde{R}_{n,z}^{-1/2} \tilde{R}_{n,z}^{-1/2} \). From Eq. (27), we find that \( \Pi_{U_1} P^T = \tilde{M} \tilde{R}_{n,z} \tilde{M} \), which corresponds to \( \Sigma = \tilde{R}_{n,z} \). Result 2 then follows from the identities (28) and (29). The corresponding proof for PI-MOESP is shown in an analogous manner. □

Another interesting result applicable to EIV identification (i.e. \( f(t) \neq 0 \)) is:

Result 3. Suppose that \( W_K = I \), and that the instruments are chosen as \( Z = P \). Then the SIV subspace estimate coincide with the subspace estimate proposed by Chou and Verhaegen (1997).

Proof. The subspace estimate computed in Chou and Verhaegen (1997) is obtained from the \( n \) dominant left singular vectors of the matrix \( \tilde{R}_{22} \), where the following QR-factorization is computed

\[
\begin{bmatrix}
U_t \\
Y_t
\end{bmatrix} P^T =
\begin{bmatrix}
\tilde{R}_{21} & 0 \\
\tilde{R}_{21} & \tilde{R}_{22}
\end{bmatrix}
\begin{bmatrix}
\tilde{Q}_1^T \\
\tilde{Q}_2^T
\end{bmatrix}. \tag{31}
\]

From Eq. (31) it is clear that \( \tilde{R}_{n,z} \Pi_{U_1} = \tilde{R}_{22} \tilde{Q}_1^T \). The left singular vectors of \( \tilde{R}_{22} \) thus coincide with those of \( \tilde{R}_{n,z} \Pi_{U_1} \), and the proof is complete. □

Comparing the weightings in Results 2 and 3, it is logical to modify the instruments as

\[
Z = \tilde{R}_{n,p}^{-1/2} P. \tag{32}
\]

This modification of Chou’s and Verhaegen’s original method will be investigated in Section 5 using a numerical example.

5. Numerical example

The focus has been on interpretations of subspace-based identification. The only new algorithmical result is that a weighting applicable to the EIV problem has been proposed. Therefore, we report only on a single numerical example illustrating the benefits of the weighting (32). The estimate of the extended observability matrix is obtained from the \( n \) principal left singular vectors of the matrix \( \tilde{R}_{n,z} \Pi_{U_1} \), where \( Z = W_p P \), and two different choices of the pre-whitening matrix \( W \) are studied:

Alg1: \( W_p = I \), as in Chou and Verhaegen (1997).
Alg2: \( W_p = \tilde{R}_{n,p}^{-1/2} \).

In the studied example, the noise-free input signal is generated as a sum of 30 sinusoids with frequencies linearly spaced in the interval \([0.1\pi, 0.6\pi]\). The output \( \tilde{y}(t) \) is generated as

\[
(1 - q^{-1} + 0.5q^{-2})\tilde{y}(t) = (1 - 0.5q^{-2})\tilde{u}(t) + (1 + q^{-1} + 0.5q^{-2})w(t), \tag{33}
\]

where \( w(t) \) is a white zero mean Gaussian random process with variance \( \sigma_w^2 = 0.1 \). The measurement noise \( f(t) \) is defined as \( f(t) = 0.5w(t) \), and \( \tilde{v}(t) \equiv 0 \). The noise free input signal is scaled so that \( 10 \log(\sigma_u/\sigma_w) = 5 \text{ dB} \). The estimate \( \hat{C} \) is obtained from the first row of \( \hat{Q}_1 \), and \( \hat{A} = \hat{Y}' \hat{Y} \). Here, \( \hat{Y} \) denotes the matrix constructed of the first \( x - 1 \) rows of \( \hat{Q}_1 \), and \( \hat{Y}' \) contains the last \( x - 1 \) rows of \( \hat{Q}_1 \). To estimate \( B \) and \( D \), the least squares solution of Eq. (23) is computed.

The outcome of this experiment is shown in Fig. 2. Here the empirical pole standard deviation is studied together with the figure of merit \( V(N) \),

\[
V(N) = \frac{1}{M} \sum_{k=1}^{M} ||\hat{p}_k - \theta_0||. \tag{34}
\]

Here, \( M = 200 \) denotes the number of independent Monte Carlo simulations, and \( \hat{p}_k \) contains all estimated parameters of the \( k \)th estimate of the transfer function from \( \tilde{u} \) to \( \tilde{y} \), when \( N \) samples are used.

From Fig. 2, it is clear that the proposed weighting (32) increases the accuracy compared with the original EIV algorithm of Chou and Verhaegen (1997).

6. Conclusions

A novel framework for subspace-based identification has been presented, and connections with existing
IV interpretation is that we found an improvement of an existing subspace method applicable to errors-in-variables identification.

References


Tony Gustafsson was born in Värnamo, Sweden, in 1969. He received the M.S. degree in electrical engineering from Chalmers University of Technology, Sweden, in 1994, and in 1999 he received the Ph.D. degree in signal processing from the same university. From 1999 to 2000, he was a postdoctoral researcher at University of California San Diego. Presently he is a research engineer at SwitchCore corporation, Göteborg, Sweden.