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Well-Distributed Measurable Sets

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## NOTES

EDITED BY SHELDON AXLER, KENNETH R. REBMAN, AND J. ARTHUR SEEBACH, JR.

*Material for this department should be sent to Professor J. Arthur Seebach, Jr., Department of Mathematics, St. Olaf College, Northfield, NM 55057.*

### WELL-DISTRIBUTED MEASURABLE SETS

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**THEOREM.** *There is a measurable set  $A \subset I = [0, 1]$  such that*

$$0 < m(A \cap V) < m(V)$$

*for every nonempty open set  $V \subset I$ .*

*Proof.* Let CTDP mean: Compact Totally Disconnected subset of  $I$ , having Positive measure.

Let  $\{I_n\}$  be an enumeration of all segments in  $I$  whose endpoints are rational. Construct sequences  $\{A_n\}, \{B_n\}$  of CTDP's as follows:

Start with disjoint CTDP's  $A_1$  and  $B_1$  in  $I_1$ .

Once  $A_1, B_1, \dots, A_{n-1}, B_{n-1}$  are chosen, their union  $C_n$  is CTD, hence  $I_n \setminus C_n$  contains a

nonempty segment  $J$ , and  $J$  contains a pair  $A_n, B_n$  of disjoint CTDP's. Continue in this way, and put

$$A = \bigcup_{n=1}^{\infty} A_n.$$

If  $V \subset I$  is open and nonempty, then  $I_n \subset V$  for some  $n$ , hence  $A_n \subset V$  and  $B_n \subset V$ . Thus

$$0 < m(A_n) \leq m(A \cap V) < m(A \cap V) + m(B_n) \leq m(V);$$

the last inequality holds because  $A$  and  $B_n$  are disjoint. Done.

The point of publishing this is to show that the highly computational construction of such a set in [1] is much more complicated than necessary.

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#### ANY QUESTIONS?

DESMOND MACHALE

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Recently, I attended a mathematical lecture given by a guest speaker where absolutely nobody, except possibly the speaker, had the remotest idea what was going on. Normally, one can absorb at least some of the preliminary definitions and follow, say, the first blackboard full of development of the theory, but on this occasion everyone was completely lost after the first definition. After the speaker had finished over an hour later to an enthusiastic round of applause, the chairman asked for questions, and, of course, there was a deathly and highly embarrassing silence. Then and there I resolved to put together a collection of universal questions for use in such situations. Such questions must sound sensible, but they are designed to cover up the total ignorance of the questioner rather than to elicit information from the speaker. The following is the list I came up with.

1. Can you produce a series of counterexamples to show that if any of the conditions of the main theorem are dropped or weakened, then the theorem no longer holds?  
[The speaker can almost always do so—if not you may have presented him with a stronger theorem!]
2. What inadequacies of the classical treatment of this subject are now becoming obvious?
3. Can your results be unified and generalized by expressing them in the language of Category Theory?  
[The answer to this question is always NO!]
4. Isn't there a suggestion of Theorem 3 in an early paper of Gauss?  
[The answer to this question is almost always YES!]
5. Isn't the constant 4.15 in Theorem 2 suspiciously close to  $4\pi/3$ ?  
[This question can clearly be generalized for any constant  $k$ —“Isn't  $k$  suspiciously close to  $(p/q)\pi$  (for suitable integers  $p$  and  $q$ )?”]
6. I'm not sure I understand the proof of Lemma 3—could you outline it for us again?  
[Lemma 3 should be just a little nontrivial, yet not more than one third of a blackboard in length.]
7. Are you familiar with a joint paper of Besovik and Bombialdi which might explain why the converse of Theorem 5 is false without further assumptions?  
[This is a dangerous question to ask unless you like living dangerously. The answer is always “NO” unless the speaker is playing the same game as you are, because Besovik and