$\boxed{1.1:3}$

$u(x,y) = \log(x^2+y^2)$

$$u_{xx} = \frac{\partial^2}{\partial x^2} u(x,y) = \frac{\partial}{\partial x} \frac{2x}{x^2+y^2} = \frac{2}{x^2+y^2} - 2x \frac{2x}{(x^2+y^2)^2} = 2\frac{y^2-x^2}{(x^2+y^2)^2}$$

$$u_{yy} = \text{"same same, different name"} = 2\frac{x^2-y^2}{(x^2+y^2)^2} = -u_{xx}$$

$$\Updownarrow$$

$$u_{xx} + u_{yy} = 0$$

$\boxed{1.2:5c}$

$$\varphi_n(x,y) = \sin(n\pi x)\sinh(\pi n y) \qquad n \in \{1,2,3,\dots\}$$

$$\frac{\partial^2}{\partial x^2}\varphi_n = -n^2\pi^2 \sin(n\pi x)\sinh(n\pi y)$$

$$\frac{\partial^2}{\partial y^2}\varphi_n = n^2\pi^2 \sin(n\pi x)\sinh(n\pi y)$$

$$\Longrightarrow \frac{\partial^2}{\partial x^2}\varphi_n + \frac{\partial^2}{\partial y^2}\varphi_n = 0$$

$\varphi_n(0,y) = 0$

$\varphi_n(1,y) = \sin(n\pi)\sinh(n\pi y) = 0$

$\varphi_n(x,0) = 0$ since $\sinh(0) = 0$.

$\boxed{1.3:\ 1}$

(a) $y u_{xx} + u_y = 0$

$$u = \xi(x)\eta(y) \quad \Rightarrow \begin{cases} u_y = \xi(x)\eta'(y) \\ u_{xx} = \xi''(x)\eta(y) \end{cases}$$

$$\left(y\,\xi''(x)\,\eta(y) + \xi(x)\,\eta'(y) = 0 \right) \frac{1}{\xi(x)\,\eta(y)\,y}$$

$$\frac{\xi''(x)}{\xi(x)} + \underbrace{\frac{\eta'(y)}{y\,\eta(y)}}_{=-\lambda} = 0$$

gives

$$\xi''(x) - \lambda\,\xi(x) = 0$$

$$\eta'(y) + \lambda\,y\,\eta(y) = 0$$

(b) $x^2 u_{xx} + x u_x + u_{yy} + u = 0$

$$u(x,y) = \xi(x)\,\eta(y)$$

$$x^2\,\xi''(x)\,\eta(y) + x\,\xi'(x)\,\eta(y) + \xi(x)\,\eta''(y) + \xi(x)\eta(y) = 0$$

$$\frac{x^2\,\xi''(x)}{\xi(x)} + x\,\frac{\xi'(x)}{\xi(x)} + \frac{\eta''(y)}{\eta(y)} + 1 = 0$$

$$\underbrace{x^2\,\frac{\xi''(x)}{\xi(x)} + x\,\frac{\xi'(x)}{\xi(x)}}_{\text{Does not depend on } y.} = \underbrace{-\frac{\eta''(y)}{\eta(y)} - 1}_{\text{Does not depend on } x.} = -\lambda$$

$$x^2 \xi''(x) + x\,\xi'(x) + \lambda\,\xi(x) = 0 \qquad \text{(Euler equation)}$$

$$\eta''(y) + (1-\lambda)\,\eta(y) = 0$$

(c) $$u_{xx} + u_{xy} + u_{yy} = 0$$

"Mixed derivative"

$$u(x,y) = \xi(x)\,\eta(y)$$

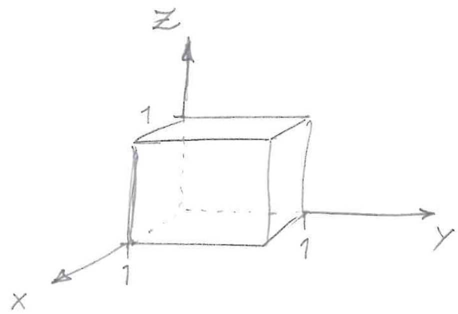
$$\frac{\xi''(x)}{\xi(x)} + \frac{\xi'(x)\,\eta'(y)}{\xi(x)\,\eta(y)} + \frac{\eta''(y)}{\eta(y)} = 0$$

(d) $$u_{xx} + u_{xy} + u_y = 0$$

$$u = \xi\eta \quad \Rightarrow \quad \xi''(x)\eta(y) + \xi'(x)\eta'(y) + \xi(x)\eta'(y) = 0$$

$\boxed{1.3:\ 6}$

$$\begin{cases} \nabla^2 u = 0, \quad u = u(x,y,z) \\ u(0,y,z) = u(1,y,z) = 0 \\ u_y(x,0,z) = u_y(x,1,z) = 0 \end{cases}$$



$$u(x,y,z) = \xi(x)\eta(y)\zeta(z)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\frac{\xi''(x)}{\xi(x)} + \frac{\eta''(y)}{\eta(y)} + \frac{\zeta''(z)}{\zeta(z)} = 0$$

$$\begin{cases} \xi''(x) + \lambda\,\xi(x) = 0 \\ \eta''(y) + \mu\,\eta(y) = 0 \\ \zeta''(z) - (\lambda+\mu)\,\zeta(z) = 0 \end{cases}$$

Boundary conditions:

$$\begin{cases} \xi(0)\eta(y)\zeta(z) = \xi(1)\eta(y)\zeta(z) = 0 \\ \xi(x)\eta'(0)\zeta(z) = \xi(x)\eta'(1)\zeta(z) = 0 \end{cases}$$

$$\begin{cases} \xi(0) = \xi(1) = 0 \\ \eta'(0) = \eta'(1) = 0 \end{cases}$$

$$\xi''(x) + \lambda\,\xi(x) = 0 \qquad \xi(x) = Ae^{i\sqrt{\lambda}x} + Be^{-i\sqrt{\lambda}x} \qquad (\lambda > 0)$$

$$0 = \xi(0) = A + B = \xi(1) = Ae^{i\sqrt{\lambda}} + Be^{-i\sqrt{\lambda}}$$

$$\eta''(y) + \mu\,\eta(y) = 0 \qquad \eta(y) = Ce^{i\sqrt{\mu}y} + De^{-i\sqrt{\mu}\cdot y} \qquad (\mu > 0)$$

$$0 = \eta'(0) = i\sqrt{\mu}C - i\sqrt{\mu}D = \eta'(1) = i\sqrt{\mu}Ce^{i\sqrt{\mu}} - i\sqrt{\mu}De^{-i\sqrt{\mu}}$$

$$\begin{cases} 0 = A\left(e^{i\sqrt{\lambda}} - e^{-i\sqrt{\lambda}}\right) = 2iA\sin(\sqrt{\lambda}) \\ 0 = i\sqrt{\mu}D\left(e^{i\sqrt{\mu}} - e^{-i\sqrt{\mu}}\right) = -2\sqrt{\mu}D\sin(\sqrt{\mu}) \end{cases}$$

$$\sqrt{\lambda} = \pi n, \quad n \in \mathbb{Z}$$

$$\sqrt{\mu} = \pi m, \quad m \in \mathbb{Z}$$

$$\xi(x) = A\left(e^{i\pi n x} - e^{-i\pi n x}\right) = 2iA \sin(\pi n x)$$

$$\eta(y) = C\left(e^{i\pi m y} + e^{-i\pi m y}\right) = 2C \cos(\pi m y)$$

$$\zeta''(z) - \pi^2(n^2+m^2)\,\zeta(z) = 0$$

$$\zeta(z) = \tilde{\alpha} e^{\pi\sqrt{n^2+m^2}\,z} + \tilde{\beta} e^{-\pi\sqrt{n^2+m^2}\,z}$$

$$u(x,y,z) = \xi(x)\,\eta(y)\,\zeta(z) =$$

$$= \alpha \sin(\pi n x)\cos(\pi m y)\, e^{\pi\sqrt{n^2+m^2}\,z} +$$

$$+ \beta \sin(\pi n x)\cos(\pi m y)\, e^{-\pi\sqrt{n^2+m^2}\,z}$$