

Föreläsning 5, del b

Deriveringsregler:

$$D e^x = e^x$$

$$\begin{aligned} D a^x &= D((e^{\ln a})^x) = D(e^{(\ln a)x}) = \\ &= (\ln a) e^{(\ln a)x} = (\ln a)(e^{\ln a})^x = (\ln a) a^x \end{aligned}$$

\uparrow inre derivata $(a > 0)$

$$D \sin x = \cos x$$

$$D \cos x = -\sin x$$

$$\begin{aligned} D \tan x &= D \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \\ &= \frac{1}{\cos^2 x} = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x \end{aligned}$$

$$\begin{aligned} D \cot x &= D \frac{\cos x}{\sin x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \\ &= -\frac{1}{\sin^2 x} = -1 - \frac{\cos^2 x}{\sin^2 x} = -1 - \cot^2 x \end{aligned}$$

Motsvarande integreringsregler:

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int (1 + \tan^2 x) dx = \tan x + C \Leftrightarrow (\text{linearitet!})$$

$$\int dx + \int \tan^2 x dx = \tan x + C \Leftrightarrow$$

$$\int \tan^2 x dx = \tan x - x + C$$

$$\int (1 + \cot^2 x) dx = -\cot x + C \Leftrightarrow \dots \Leftrightarrow$$

$$\int \cot^2 x dx = -\cot x - x + C$$