

Föreläsning 8, del e

2.10 Beräkna med hjälp av polynomdivision:

a) $\int \frac{x}{x-1} dx$

Lösning:

$$\frac{x}{x-1} = \frac{x-1+1}{x-1} = \frac{x-1}{x-1} + \frac{1}{x-1} = 1 + \frac{1}{x-1}$$

Med uppställning:

$$\begin{array}{r} 1 \\ x-1 \overline{) x} \\ \underline{-(x-1)} \\ 1 \end{array} \quad \frac{x}{x-1} = 1 + \frac{1}{x-1}$$

$$\begin{aligned} \int \frac{x}{x-1} dx &= \int \left(1 + \frac{1}{x-1} \right) dx = \\ &= \int dx + \int \frac{dx}{x-1} = x + \ln|x-1| + C \end{aligned}$$

Svar: $\int \frac{x}{x-1} dx = x + \ln|x-1| + C$

b) $\int \frac{x^2}{x-1} dx$

Lösning: (konjugatregeln)

$$\begin{aligned} \frac{x^2}{x-1} &= \frac{x^2-1+1}{x-1} = \frac{(x+1)(x-1)+1}{x-1} = \\ &= \frac{(x+1)(x-1)}{x-1} + \frac{1}{x-1} = x+1 + \frac{1}{x-1} \end{aligned}$$

Med uppställning:

$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2} \\ \underline{-(x^2-x)} \\ x \\ \underline{-(x-1)} \\ 1 \end{array} \quad \frac{x^2}{x-1} = x+1 + \frac{1}{x-1}$$

$$\begin{aligned} \int \frac{x^2}{x-1} dx &= \int \left(x+1 + \frac{1}{x-1} \right) dx = \\ &= \int x dx + \int dx + \int \frac{dx}{x-1} = \\ &= \frac{1}{2} x^2 + x + \ln|x-1| + C \end{aligned}$$

Svar: $\int \frac{x^2}{x-1} dx = \frac{1}{2} x^2 + x + \ln|x-1| + C$