1 Theoretical questions to examination.

1. Give an example of a system of ODEs in $\mathbb{R}^2$ having some solutions $\varphi(t,\xi)$ that do not have ω or α limit sets.

   i) The simplest example is a system that has all trajectories tending to infinity with $t \to \infty$

   ii) An equation with bounded domain an all solutions having $\sup I_\xi < \infty$ (here $I_\xi$is the maximal interval for initial point ξ) and therefore tending to it's boundary with $t \to \sup I_\xi$.

2. Show that the ω - limit set Ω_ξ for solutions $\varphi(t,\xi)$ having closure of the orbit $O_+(\xi)$ compact, must be non-empty.

   Hint: use the key property of compact sets that any bounded sequence in a compact set has a convergent subsequece.

3. Show that for Lipschitz right hand side f in the equation $x' = f(x)$ the transfer mappig $\varphi(t,\xi)$ is Lipscitz with respect to both (each of) variables.

   Hint. Write the I.V.P. in intergal form and use Grönwall's inequality!

4. Sketch a trajectory illustrating the definition of ω - limit set.

5. Assume that $0 \in G$ and is an asymptotically stable equilibrium point. Show that the domain of attraction $\mathcal{A}$ to 0 is an open set. Exercise 5.14 in L.R.

6. Suppose a monodromy matrix M is given for a periodic linear system in $\mathbb{R}^2$ with period T . Can one calculate exactly values $\varphi(t,\xi)$ of the solution with initial data ξ for $t = T$, $3T$, $5T$?

   Hint. Use the expression for the transfer matrix for periodic systems.

7. How long time it could take for a solution $\varphi(t,\xi)$ to the equation $x' = f(x)$ with $f : G \to \mathbb{R}^n$ to reach:

   a) an asymptotically stable equilibrium point

   b) the boundary ∂G of the domain G in case $\varphi(t,\xi) \to \partial G$ as t tends to $\sup(I_\xi)$.

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