Föreläsning 6, del e (demonstration testuppgifter)

2.7 e) Beräkna $\int (x^3+1)e^x\,dx$!

Lösning: (upprepad partialintegration!)

$$\int (x^3+1)e^x\,dx = \int e^x(x^3+1)\,dx =$$

$$= e^x(x^3+1) - \int e^x(3x^2)\,dx =$$

$$= e^x(x^3+1) - 3\int e^x x^2\,dx =$$

$$= e^x(x^3+1) - 3\left(e^x x^2 - \int e^x(2x)\,dx\right) =$$

$$= e^x(x^3+1) - 3e^x x^2 + 3\cdot 2\int e^x x\,dx =$$

$$= e^x(x^3+1-3x^2) + 6\left(e^x x - \int e^x\cdot 1\,dx\right) =$$

$$= e^x(x^3+1-3x^2+6x) - 6e^x + C =$$

$$= e^x(x^3+1-3x^2+6x-6) + C =$$

$$= (x^3-3x^2+6x-5)e^x + C$$

Svar: $\int (x^3+1)e^x\,dx =$

$= (x^3-3x^2+6x-5)e^x + C$

2.7 f) Beräkna $\int (\ln t)^2\,dt$!

Lösning: (upprepad partialintegration!)

$$\int (\ln t)^2\,dt = \int 1\cdot(\ln t)^2\,dt =$$

inre derivata!

$$= t(\ln t)^2 - \int t\left(\frac{d}{dt}(\ln t)^2\right)dt =$$

$$= t(\ln t)^2 - \int t\left(2(\ln t)\frac{d}{dt}\ln t\,dt\right) =$$

$$= t(\ln t)^2 - 2\int t(\ln t)\frac{1}{t}\,dt =$$

$$= t(\ln t)^2 - 2\int (\ln t)\,dt =$$

$$= t(\ln t)^2 - 2\int 1\cdot(\ln t)\,dt =$$

$$= t(\ln t)^2 - 2\left(t\ln t - \int t\frac{1}{t}\,dt\right) =$$

$$= t(\ln t)^2 - 2t\ln t + 2\int dt =$$

$$= t(\ln t)^2 - 2t\ln t + 2t + C$$

Svar: $\int (\ln t)^2\,dt =$

$= t(\ln t)^2 - 2t\ln t + 2t + C$