Fourieranalys MVE030 och Fourier Metoder MVE290 20.mars.2020
Betygsgränser: 3: 40 poäng, 4: 53 poäng, 5: 67 poäng.
Maximalt antal poäng: 80.
Hjälpmedel: BETA.
Examinator: Julie Rowlett.
Telefonvakt: Carl-Joar Karlsson 5325. OBS! Om ni är osäker på något fråga!
(If you are unsure about anything whatsoever, please ask!)

1. Solve:
$$\begin{cases} u_{tt} - u_{xx} = f(t)g(x) & 0 < t, -1 < x < 1 \\ u(x,0) = \varphi(x) = u_t(x,0) \\ u(-1,t) = 0, \quad u_x(1,t) = 5 \end{cases}$$
(10p)

2. Use the Fourier series expansion of $\cos(\alpha t)$ on $(-\pi, \pi)$ to compute for $|\alpha| < 1$
$$\prod_{n \geq 1} \frac{n^2 - \alpha^2}{n^2}.$$
Hint: The Fourier series is
$$\frac{\sin(\alpha\pi)}{\pi} \left(\frac{1}{\alpha} + 2\alpha \sum_{n \geq 1} \frac{(-1)^{n+1} \cos(nt)}{n^2 - \alpha^2} \right).$$
(10p)

3. Compute
$$\int_0^\infty \frac{\sin(\sqrt{t})}{e^{2t}} dt.$$
(10 p)

4. Solve:
$$\begin{cases} u(0,t) = f(t) & t > 0 \\ u_t(x,t) - u_{xx}(x,t) = 0 & t, x > 0 \\ u(x,0) = 0 & x > 0 \end{cases}$$
(10 p)

5. This could be a PDE in a half space with a nice boundary condition (extend evenly or oddly!). Another reasonable candidate is computing an integral with help of Fourier transform (like those EÖ number 7-12). Or an integral equation where you use convolution and Fourier transform (like EÖ 13, 14). Or an SLP (EÖ 23, 24). Or a PDE in a box (EÖ 25), or in a disk, or a wedge. Or a best approximation. (10p)

6. This could be a best approximation, or an SLP, or maybe something involving Bessel functions. Or possibly something to test conceptual understanding without actually needing to do much calculating. (10p)

7. Prove a theory item! (10p)

8. Prove a theory item! (10 p)

Fourier transformer (Fourier transforms) där $a > 0$ och $c \in \mathbb{R}$.

$f(x)$	$\hat{f}(\xi)$
$f(x-c)$	$e^{-ic\xi}\hat{f}(\xi)$
$e^{ixc}f(x)$	$\hat{f}(\xi - c)$
$f(ax)$	$a^{-1}\hat{f}(a^{-1}\xi)$
$f'(x)$	$i\xi\hat{f}(\xi)$
$xf(x)$	$i(\hat{f})'(\xi)$
$(f * g)(x)$	$\hat{f}(\xi)\hat{g}(\xi)$
$f(x)g(x)$	$(2\pi)^{-1}(\hat{f} * \hat{g})(\xi)$
$e^{-ax^2/2}$	$\sqrt{2\pi/a}e^{-\xi^2/(2a)}$
$(x^2+a^2)^{-1}$	$(\pi/a)e^{-a\|\xi\|}$
$e^{-a\|x\|}$	$2a(\xi^2+a^2)^{-1}$
$\chi_a(x) = \begin{cases} 1 & \|x\| < a \\ 0 & \|x\| > a \end{cases}$	$2\xi^{-1}\sin(a\xi)$
$x^{-1}\sin(ax)$	$\pi\chi_a(\xi) = \begin{cases} \pi & \|\xi\| < a \\ 0 & \|\xi\| > a \end{cases}$

Laplace transformer (Laplace transforms) där $a > 0$, $c \in \mathbb{C}$, och

$$H(t) := \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

$H(t)f(t)$	$\widetilde{f}(z)$
$H(t-a)f(t-a)$	$e^{-az}\widetilde{f}(z)$
$H(t)e^{ct}f(t)$	$\widetilde{f}(z-c)$
$H(t)f(at)$	$a^{-1}\widetilde{f}(a^{-1}z)$
$H(t)f'(t)$	$z\widetilde{f}(z) - f(0)$
$H(t)\int_0^t f(s)ds$	$z^{-1}\widetilde{f}(z)$
$H(t)(f*g)(t)$	$\widetilde{f}(z)\widetilde{g}(z)$
$H(t)t^{-1/2}e^{-a^2/(4t)}$	$\sqrt{\pi/z}e^{-a\sqrt{z}}$
$H(t)t^{-3/2}e^{-a^2/(4t)}$	$2a^{-1}\sqrt{\pi}e^{-a\sqrt{z}}$
$H(t)J_0(\sqrt{t})$	$z^{-1}e^{-1/(4z)}$
$H(t)\sin(ct)$	$c/(z^2+c^2)$
$H(t)\cos(ct)$	$z/(z^2+c^2)$
$H(t)e^{-a^2t^2}$	$(\sqrt{\pi}/(2a))e^{z^2/(4a^2)}\operatorname{erfc}(z/(2a))$
$H(t)\sin(\sqrt{at})$	$\sqrt{\pi a/(4z^3)}e^{-a/(4z)}$

Lycka till! May the force be with you! ♡ Julie Rowlett, Carl-Joar Karlsson, Joao Pedro Paulos, Erik Jansson, Kolya Pochekai