Exercises in Credit Risk: Extreme Value Statistics

MVE220/MSA400

April 6, 2022

During the exercise session 7/4 2022 we will consider the following problems from [1].

1. Consider a static credit portfolio with 1000 obligors which we model as mixed binomial model inspired by the Merton framework. The individual one-year default probability is $\bar{p} = 5\%$, the individual loss is $\ell = 60\%$, the default correlation is $\rho = 25\%$ and each loan has notional 1 million SEK. Use the large portfolio approximation (LPA) formula to compute the probability that within one year, the total portfolio credit loss will be more than 50 million SEK but less than 100 million SEK.

2. Consider a static credit portfolio with $m = 1000$ obligors which we model as mixed binomial model inspired by the Merton framework. The individual one-year default probability is $\bar{p}$, the individual loss is $\ell = 60\%$, the default correlation is $\rho = 12\%$ and each loan has notional 1 million SEK. We also know that the probability that within one year, the total portfolio credit loss will be less than 20 million SEK is 53.5%. Use the LPA-approximation formula to compute the probability that within one year, the total portfolio credit loss will be more than 50 million SEK but less than 100 million SEK.

3. Consider a static credit portfolio with $m = 1000$ obligors which we model as mixed binomial model with a logit-normal mixing distribution (for one year, say) and where each loan has notional 1 million SEK and the individual loss is $\ell = 60\%$. We know that the one-year LPA-VaR formula produces the values $\text{VaR}_{95\%}(L) = 116.9$ million SEK and $\text{VaR}_{99\%}(L) = 254.2$ million SEK. Given this, compute the one-year $\text{VaR}_\alpha(L)$ for $\alpha = 99.9\%$ with the LPA-VaR formula.

4. Consider a static credit portfolio with m obligors. We model this portfolio as a mixed binomial model with a one-year default mixing distribution $p(Z) = \frac{Z}{K}$ where Z is a binomially distributed random variable, $Z \sim \text{Bin}(K, q)$ for $0 < q < 1$ and K is an integer such that $K \geq 2$. If $m = 10$, $K = 3$ and $q = 0.05$ compute the probability of having no defaults in this portfolio within one year.

5. Consider a static credit portfolio with m obligors. We model this portfolio as a mixed binomial model with a one-year default mixing distribution $p(Z) = \frac{Z}{K}$ where Z is a binomially distributed random variable, $Z \sim \text{Bin}(K, q)$ for $0 < q < 1$ and K is an integer such that $K \geq 2$. If $m = 100$, $K = 5$ and $q = 0.05$, compute the pairwise default correlation $\rho_X = \text{Corr}(X_i, X_j)$ for $i \neq j$.

6. Consider a static credit portfolio with m obligors which we model as mixed beta binomial model with parameter a and b for one year, that is the one-year conditional default probability $p(Z)$ is given by $p(Z) = P[X_i = 1|Z] = Z$ where $Z \sim \text{Beta}(a, b)$. Let $a = 2$ and $E[p(Z)] = 0.1$. Compute the pairwise default correlation $\rho_X = \text{Corr}(X_i, X_j)$ for $i \neq j$.

References

[1] Alexander Herbertsson. Static credit portfolio models lecture notes, April 2019.