Föreläsning 14, del f (demonstration testuppgifter)

5.2b (forts.)

$$\frac{1}{K}\left(\frac{1}{P(t)}+\frac{1}{K-P(t)}\right)P'(t)=\frac{r}{K}$$

$$\Leftrightarrow \left(\frac{1}{P(t)}+\frac{1}{K-P(t)}\right)P'(t)=r$$

$$\Leftrightarrow \frac{d}{dt}\Big(\ln P(t)-\ln\big(K-P(t)\big)\Big)=r$$

integrera med avseende på t!

$$\Leftrightarrow \ln P(t)-\ln\big(K-P(t)\big)=rt+C$$

$$\Leftrightarrow \ln\frac{P(t)}{K-P(t)}=rt+C$$

exponentiera!

$$\Leftrightarrow \frac{P(t)}{K-P(t)}=e^{rt}e^{C}$$

invertera!

$$\Leftrightarrow \frac{K-P(t)}{P(t)}=e^{-C}e^{-rt}$$

$$\Leftrightarrow \frac{K}{P(t)}-1=e^{-C}e^{-rt} \Leftrightarrow \frac{K}{P(t)}=e^{-C}e^{-rt}+1$$

$$\Leftrightarrow P(t)=\frac{K}{1+e^{-C}e^{-rt}}$$

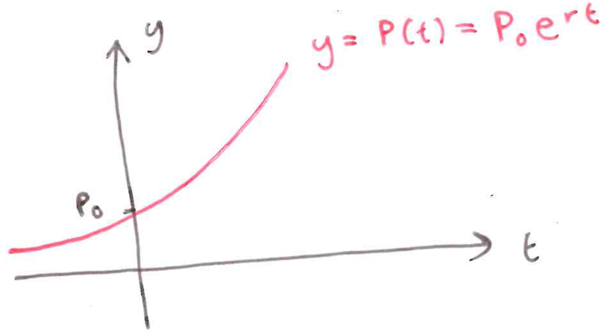
Vi bestämmer konstanten C med hjälp av begynnelsevillkoret:

$$P(0)=P_0 \Leftrightarrow P_0=\frac{K}{1+e^{-C}} \Leftrightarrow$$

$$1+e^{-C}=\frac{K}{P_0} \Leftrightarrow e^{-C}=\frac{K}{P_0}-1$$

Svar: $P(t)=\dfrac{K}{1+\left(\frac{K}{P_0}-1\right)e^{-rt}} \to K$ då $K\to\infty$

a)



b)