Going from the primal tableau to the dual tableau

I will do this for the system in exercise 4.23 in K-B. Assume that we have reached the tableau:

\[
\begin{array}{cccccc}
X_1 & X_2 & X_3 & X_4 & X_5 & \\
0 & 1 & 1/2 & -1/2 & 0 & 3 \\
1 & 0 & -1/4 & 3/4 & 0 & 3/2 \\
0 & 0 & -3/4 & -3/4 & 1 & -1/2 \\
0 & 0 & 1/4 & 1/4 & 0 & 9/2 \\
\end{array}
\]

\(X_1, X_2, X_5\) basic vars for \((P)\)
\(X_3, X_4\) nonbasic vars for \((P)\)
\(\Rightarrow y_1, y_2, y_5\) nonbasic for \((D)\)
\(y_3, y_4\) basic for \((D)\).

Start by labelling the rows by the basic vars \(y_3, y_4\).

In the tableau, there has to be a 1 in the \(y_3 - y_5\) position and also in the \(y_4 - y_4\) pos. Put zeros in the other places in the columns corr. to the basic vars.

Next, put in the entries for the nonbasic vars. The entry for row \(y_3\), column \(y_1\), for example, should be the same as the one in row \(X_3\), column \(x_3\) in the primal tableau, but with the opposite sign.

\[
\begin{array}{cccccc}
y_1 & y_2 & y_3 & y_4 & y_5 & \\
y_3 & 1/4 & -1/2 & 1 & 0 & 3/4 & 1/4 \\
y_4 & -3/4 & 1/2 & 0 & 1 & 3/4 & 1/4 \\
\end{array}
\]

After this, we can fill in the entries for the objective row and the RHS. and also the objective value (N.B. note the sign!)

Why does this work?
See next page!
To see how we get this tableau for the dual system, we write down the two systems on standard form:

\[
\begin{align*}
\text{(P)} \quad \frac{1}{2} x_3 - \frac{1}{2} x_4 & \leq 3 \quad \text{(slack variable } x_2) \\
-\frac{1}{4} x_3 + \frac{3}{4} x_4 & \leq 3/2 \quad \text{(slack variable } x_1) \\
-\frac{3}{4} x_3 - \frac{3}{4} x_4 & \leq -1/2 \quad \text{(slack variable } x_5) \\
\end{align*}
\]

Objective function: \( -\frac{1}{4} x_3 + \frac{1}{4} x_4 + \frac{9}{2} \cdot x_j \geq 2 \quad j = 1, \ldots, 5 \)

When choosing \( y_j \) variables for the dual problem, use \( y_j \) to be the variable corresponding to the second constraint, so that the number of \( y_j \) corresponds to the slack variable \( x_j \) that was removed.

The dual problem is then:

\[
\begin{align*}
\text{minimize } & -v = \frac{3}{2} y_1 + 3 y_2 - \frac{1}{2} y_5 + \frac{9}{2} \\
\text{subject to } & \begin{cases}
-\frac{1}{4} y_1 + \frac{1}{2} y_2 - \frac{3}{4} y_5 \geq -\frac{1}{4} \\
\frac{3}{4} y_1 - \frac{1}{2} y_2 - \frac{3}{4} y_5 \geq -\frac{1}{4} \\
y_j \geq 0, \quad j = 1, \ldots, 5.
\end{cases}
\end{align*}
\]

Write as a maximization problem:

\[
\begin{align*}
\text{maximize } & v = -\frac{3}{2} y_1 - 3 y_2 + \frac{1}{2} y_5 - \frac{9}{2} \\
\text{subject to } & \begin{cases}
\frac{1}{4} y_1 - \frac{1}{2} y_2 + \frac{3}{4} y_5 \leq +\frac{1}{4} \\
-\frac{3}{4} y_1 + \frac{1}{2} y_2 + \frac{3}{4} y_5 \leq +\frac{1}{4} \\
y_j \geq 0, \quad j = 1, \ldots, 5.
\end{cases}
\end{align*}
\]

Introduce slack vars, you get the last tableau on the previous page.