Solutions to the exercises 1) and 2) should be correct, well described and motivated. Each exercise should have a solution starting on a new sheet of paper.

1) The stationary discrete time process \( Y_t, t = 0, \pm 1, \pm 2, \ldots \), is the output of the causal linear filter with the impulse response

\[
 h(t) = \begin{cases} 
 1 & t = 0, \\
 0.1(-0.7)^{-1} & t \geq 1. 
\end{cases}
\]

The input process \( X_t, t = 0, \pm 1, \pm 2, \ldots \), is a zero-mean Gaussian white noise sequence with \( \mathbb{V}[X_t] = 1 \).

a) Determine the cross-covariance \( r_{XY}(\tau) \) and the cross-spectrum \( R_{XY}(f) \) between the input and output processes.

b) Determine the covariance function \( r_Y(\tau) \) for the output process.

2) In connection to edge detection we model the edge as a continuous time step signal

\[
 s(t) = \begin{cases} 
 0 & t < 0, \\
 1 & t \geq 0. 
\end{cases}
\]

The step signal is disturbed by a stationary Gaussian process \( X(t) \) with expected value \( m_X = 0 \) and covariance function \( r_X(\tau) = e^{-r^2} \). The process \( Z(t) = s(t) + X(t) \) is filtered through a filter with impulse response

\[
 h(t) = e^{-\frac{t^2}{2g^2}}, \quad -\infty < t < \infty,
\]

giving the output process \( Y(t) \). To be able to detect the edge \( s(t) \) it is interesting to differentiate \( Y(t) \). Compute

\[
 \frac{V[Y'(0)]}{E[Y(0)^2]},
\]

as a function of the parameter \( g \).