This worksheet explains what we are going to cover this week, and is meant to help you plan how to work with the material. Please consult the first worksheet for more detailed explanations.

**Schedule:** Note that we have dropped the Q&A sessions as these were hardly used.

<table>
<thead>
<tr>
<th>Schedule</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>08:15-10:00</td>
<td>SI Linear Algebra</td>
<td>SI Linear Algebra</td>
<td>Problem seminar Analysis</td>
<td>Problem seminar Analysis</td>
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<td>10:15-12:00</td>
<td>Lecture Linear Algebra</td>
<td>Lecture Analysis</td>
<td>Problem seminar Analysis</td>
<td>Lecture Linear Algebra</td>
<td>SI Analysis</td>
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<td>12:00-13:00</td>
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<td>SI Analysis</td>
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<tr>
<td>13:15-15:00</td>
<td>Problem seminar</td>
<td>Problem seminar</td>
<td>Linear Algebra</td>
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<td>15:15-17:00</td>
<td>SI Analysis</td>
<td>SI Analysis</td>
<td>Lecture Analysis</td>
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</table>

**Rotating schedule:** As an extra precaution, we choose to use invitations this week also. Please confirm your invitation for next week at the latest by Sunday by clicking here.

- **Tuesday:** Mentor groups 1 – 8 (58 invited students)
- **Wednesday:** Mentor groups 9 – 16 and 201 (57 invited students)
- **Friday:** Mentor groups 1 – 8 (58 invited students)

**Obligatory activities related to this week:**

- **Confirm invitation to class.** Deadline is Sunday 4 October.
- **Group-wise presentation of problems in problem seminar.** See next page for details.
- **Mentor meeting.** Themes for the meeting this week is to discuss the feedback on homework 2, as well as the newly published homework 3. Also, you may want to schedule the meeting ahead of your presentation if you have one...
- **Homework:** Homework 3 is published this Monday. For the students who did not pass homework 2, the deadline for resubmission is on Friday.
• **The big picture:** I have taken away the "what we do carefully" since this is essentially covered by connecting the dots between the other two points.

  - **Chapter 7.1**
    * **Main take-aways:** The definition of the derivative, and the geometric intuition behind it.
    * **What we are less careful about:** Well nothing, but I can remark that it is perhaps not super clear is that the concept of a tangent line is actually also defined here. That is, if, say, \( f'(a) \) exists, then the line from Remark 7.2, with \( k = f'(a) \), is by definition the tangent line of \( y = f(x) \) at \( x = a \). Also, Definition 7.5 should mention that at endpoints, the definition of the derivative is defined using a one-sided limit.

  - **Chapter 7.2**
    * **Main take-aways:** How to apply the derivative on examples.
    * **What we are less careful about:** Note that you are supposed to fill out Proposition 7.13 even though we haven’t proved these formulas yet (use Google if you want – we prove most/all of them later in the chapter). Also, you should rename the first column to "Your favourite primitive of \( f \)" to avoid having to put \(+C\) everywhere.

  - **Chapter 7.3**
    * **Main take-aways:** That the proofs for the rules of differentiation are just exercises on the use of the rulebook for the limit of functions. Also, please notice that differentiability implies continuity is a "detail" that we frequently need to invoke.
    * **What we are less careful about:** I think all is good, except that that I just discovered an annoying error in the "correct" proof of the chain rule (I supply a correct proof in the film on this – so a challenge is to try to figure out why this proof also has a flaw... now, both of the flawed proofs can be fixed by adding extra assumptions. So, here, another nice exercise would be to identify these, and then try to "judge" which of these arguments, when fixed in this way, is better).

  - **Chapter 7.4**
    * **Main take-aways:** How we obtain the differentiation rules for the "usual" functions. In particular, the use of implicit differentiation to find the differentiation formulas of our most famous inverse functions is kind of neat.
    * **What we are less careful about:** Well, here we are using the geometric definitions for the logarithm and the trigonometric functions, so this means we allow ourselves to "see" certain properties. Moreover, we use the implicit function theorem without any real explanation (use Google if you hunger for one). Note that our use of the implicit function theorem is actually not fundamental to the course, as we mainly use this technique as a mnemonic device to recall the formula for the derivative of inverse functions (I have provided the proof that inverse functions, under suitable assumptions, are differentiable in a film).

  - **Chapter 8.1**
    * **Main take-aways:** The formulation for the Mean Value Theorem, and its first consequence (the VERY common-sensical Corollary 8.3).
    * **What we are less careful about:** Nothing.
Tuesday 6/10:
• **Pages to read before lecture:** Do a first reading of pages 247 – 296, and 301–304.

• **Films available on YouTube:**
  - Chapter 7:
    * The definition of the derivative explained (7.3-4), 6:41 min.
    * How to use the derivative on examples, part 1 (7.9), 5:22 min.
    * How to use the derivative on examples, part 2 (7.15), 5:17 min.
    * How to use the derivative on examples, part 3 (7.16), 2:14 min.
    * How to use the chain rule, part 1 (7.20), 4:20 min.
    * How to use the chain rule, part 2 (7.24), 3:58 min.
    * Proof of the sum rule for the derivative (7.27), 2:26 min.
    * On the proof that differentiability implies continuity (7.28), 3:48 min.
    * On the proof of the product rule for the derivative (7.30), 5:11 min.
    * A fake proof of the chain rule for the derivative (7.32), 5:45 min.
    * A correct proof of the chain rule for the derivative, 16:59 min.
    * On how to prove the differentiation formula for the logarithm (7.34-36), 1:57 min.
    * On how to prove the differentiation formula for sine and cosine (7.41-46), 6:35 min.
    * A brief explanation of implicit differentiation (7.51), 6:31 min.
    * How to use implicit differentiation to differentiate inverse functions (7.54), 3:41 min.
  - Chapter 8: (these films will be delayed a little bit longer)
    * The statement of the Mean Value Theorem explained
    * A first consequence of the Mean Value Theorem

• **Reading exercises:**
  - Chapter 7.1: 7.4, 7.6, 7.10.
  - Chapter 7.2: None.
  - Chapter 7.3: 7.33.
  - Chapter 7.4: 7.41 (solved in one of the videos).
  - Chapter 8.1: None.

Wednesday 8/10:
• **Pages to read before problem session:** 247 – 286.

• **Problems to work on:**
  - 7.1: 7.7, 7.8, 7.11.
  - 7.3: 7.28, 7.29, 7.30.

• **Problems to present:** (Each group should spend 5-10 minutes in total.)
  - Mentor group 13: 7.11.
  - Mentor group 14: 7.17c+7.21c.
  - Mentor group 16: 7.26a
  - Mentor group 103: 7.28.
Friday 9/10:

- **Pages to read before problem session:** 287 – 296, 301–304.

- **Problems to work on:**
  - 7.3: 7.31, Extra: figure out why the proof on page 287 fails.
  - 7.4: **7.36**, 7.38, 7.41 (this exercise is solved in one of the films), 7.43, **7.45**, 7.46, 7.47, 7.52, 7.55, **7.56–57**, (7.58).
  - 8.1: 8.4-5, **8.6**.

- **Problems to present:** (Each group should spend 5-10 minutes in total.)
  - Mentor group 1: 7.31.
  - Mentor group 2: 7.36.
  - Mentor group 3: 7.45.
  - Mentor group 4: 7.55b.
  - Mentor group 104: 8.6.

- **How to prepare for lecture:** Look over your notes from this week, and determine what parts you understand the least. Try to formulate some questions and post them on the mentimeter page that will be made available after the problem session on Friday.

- **Modifications to lecture notes:**
  - Definition 7.5: Add that at endpoints, the definition should use one-sided limits.
  - Add a definition after Definition 7.5 that says if \( f'(a) \) exists, then the tangent line of \( y = f(x) \) at \( x = a \) is, by definition, the straight line \( y = k(x-a) + f(a) \) with \( k = f'(a) \).
  - Example 7.9: In the first line after the definition of \( f(x) \), it should say "For what values of \( C... \)."
  - Proposition 7.13: Rename first column to "Your favourite primitive of \( f \)."
  - Proposition 7.14: Add the following rule: Suppose that \( f \) is invertible, continuous and defined on an interval. If \( f' \) exists at \( x \) and is non-zero there, then
    \[
    (vi) \quad \frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}
    \]
    After the proposition, a remark should be added saying something like "The formula for the derivative of an inverse function is perhaps the most difficult one to use, but fortunately, when we learn how to differentiate implicitly, it becomes a bit easier to handle."
  - Exercise 7.25: Here, to make the exercise slightly more interesting, \( \ln(\ln x) \) should be replaced by \( \ln(\ln(\ln x)) \).
  - Page 287: Note that there is a flaw in this proof. So here, an exercise should be added asking for it. A correct proof is provided in the films.
  - Table 7.37: The title should be "Some favourite primitives of \( x \)", and the second row should be labeled "a primitive".
  - Definition 7.48: This definition should be demoted to a reminder in the text (this definition was given in an earlier worksheet).
– Proposition 7.49: The part about $\exp(0) = 1$ should be erased as it was established in a previous worksheet.

– Exercise 7.52: Here a part (d) should be added asking you to check the derivative at the point $(1/\sqrt{2}, 1/\sqrt{2})$, and then try to figure out why you get the answer that you do.

– Exercise 7.53: Here, I should point out that the implicit function theorem not only informs you of when $y$ can be considered as a function of $x$, but also when this function is differentiable.

– Example 7.54: Here $D_{\arcsin}$ and $R_{\arcsin}$ needs to be replaced by $D_{\arccos}$ and $R_{\arccos}$ (and notice that the range actually changes when you do this).

– Example 7.57: Here you should add the assumption that $f^{-1}$ is differentiable and that the derivative of $f$ is non-zero at $f^{-1}(x)$. 