Computer Vision: Lecture 4

Carl Olsson

2019-01-29
Keypoint detection and Matching.

- Repetition: DLT
- Triangulation
- Homography Estimation
- Panoramas
Algorithm for solving
\[
\min_{\|v\|^2=1} \|Mv\|^2.
\]

1. Compute the factorization
\[M = U S V^T\]
(in Matlab).

2. Select the solution
\[v = \text{last column of } V.\]

Can solve homogeneous least squares problems:
Ex. The resection problem: Find \(P\) and \(\lambda_i\)
\[\lambda_i x_i \approx P X_i \text{ for all } i.\]
Triangulation

Known

Image points $\{x_{ij}\}$.

Camera matrices $P_j$

Sought

3D points $X_i$, such that

$$\lambda_{ij}x_{ij} = P_jX_i$$
Fixed cameras: Determine one 3D point at a time.

**Problem Formulation**

Given measured projections $x_i$ and known camera matrices $P_i$, $i = 1, ..., n$ compute the corresponding scene point $X$. Solve

$$\lambda_i x_i = P_i X \quad i = 1, ..., n$$

$3n$ equations, $3 + n$ unknowns. Need $3n \geq 3 + n \Rightarrow n \geq 2$ points.
Triangulation Geometric Interpretation

Two cameras:

The 3D point is the intersection of the viewing rays.
Degenerate Configurations

If all camera centers and the unknown 3D point $X$ are on a line, $X$ cannot be uniquely determined.
Viewing rays may not intersect in 3D.

**DLT**

Find the least squares solution of

\[ \lambda_1 x_1 = P_1 X \]
\[ \lambda_2 x_2 = P_2 X \]
\[ \vdots \]

In matrix form:

\[
\begin{bmatrix}
P_1 & -x_1 & 0 & \cdots \\
P_2 & 0 & -x_2 & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
x \\
\lambda_1 \\
\lambda_2 \\
\vdots \\
\end{bmatrix}
= 0.
\]
Near Degenerate Configurations

\[
\min_{\|v\|^2=1} \|Mv\|^2 = \min_X f(X)
\]

Reduced DLT objective (with known \(X\)):

\[
f(X) = \min_{\lambda_1^2 + \lambda_2^2 + \|X\|^2 \gamma^2 = 1} \left\| M \begin{bmatrix} \gamma X \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \right\|^2.
\]
Near Degenerate Configurations

DLT estimations with noise
Problem Formulation

Given 2D points $x_i$ and corresponding points $y_i$ related by a projective transformation find $H$ such that
\[ \lambda_i y_i = H x_i, \quad i = 1, ..., N. \]

$3N$ equations, $8 + N$ unknowns
Need $3N \geq 8 + N \Rightarrow N \geq 4$ point correspondences.
Two images of a plane are related by a $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ homography.
Uncalibrated solutions to structure from motion are related by $\mathbb{P}^3 \rightarrow \mathbb{P}^3$ homographies.
In general a set of points in $\mathbb{P}^n$ are called **projectively independent** if they have homogeneous coordinates that are linearly independent as vectors in $\mathbb{R}^{n+1}$.

A set of $n + 2$ points in $\mathbb{P}^n$ is called a **projective basis** if no subset of $n + 1$ points is projectively dependent.

A projective transformation $\mathbb{P}^n \rightarrow \mathbb{P}^n$ is uniquely determined by the mapping of the $n + 2$ points of a projective basis.

**Ex1.** $\mathbb{P}^2 \rightarrow \mathbb{P}^2$: 4 points, no 3 on a line.
**Ex2.** $\mathbb{P}^3 \rightarrow \mathbb{P}^3$: 5 points, no 4 on a plane.
Degenerate Cases
Homography Estimation with Noise

**DLT**

If

\[
H = \begin{bmatrix}
H_1^T \\
H_2^T \\
H_3^T
\end{bmatrix}
\]

and

\[
y_i = \begin{bmatrix}
x_i \\
y_i \\
1
\end{bmatrix}
\]

solve

\[
\begin{bmatrix}
x_1^T & 0 & 0 & -x_1 & 0 & \ldots \\
0 & x_1^T & 0 & -y_1 & 0 & \ldots \\
0 & 0 & x_1^T & -1 & 0 & \ldots \\
x_2^T & 0 & 0 & 0 & -x_2 & \ldots \\
0 & x_2^T & 0 & 0 & -y_2 & \ldots \\
0 & 0 & x_2^T & -1 & 0 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
H_1^1 \\
H_2^2 \\
H_3^3 \\
\lambda_1 \\
\lambda_2 \\
\vdots
\end{bmatrix}
= 0.
\]
$H_{21}$ estimated from green matches, $H_{32}$ estimated from red matches.
$H_{21}$ estimated from green matches, $H_{32}$ estimated from red matches.

- Transform image 2 using $H_{21}$. 

Panoramas and Compositions

\( H_{21} \) estimated from green matches, \( H_{32} \) estimated from red matches.

- Transform image 2 using \( H_{21} \).
- Stitch.
Panoramas and Compositions

$H_{21}$ estimated from green matches, $H_{32}$ estimated from red matches.

- Transform image 2 using $H_{21}$.
- Stitch.
- Transform image 2 using $H_{31} = H_{21}H_{32}$. 
$H_{21}$ estimated from green matches, $H_{32}$ estimated from red matches.

- Transform image 2 using $H_{21}$.
- Stitch.
- Transform image 2 using $H_{31} = H_{21}H_{32}$.
- Stitch.
Panoramas and Compositions
Panoramas

For calibrated cameras:

![Diagram showing Panoramas for calibrated cameras with Image1 and Image2 labeled along the X and Z axes.](image-url)
Panoramas

For calibrated cameras:
Panoramas

Points are transformed to the first image.
Panoramas
For calibrated cameras:

Distances are not preserved. Points close to the x-axis tend to infinity.
Panoramas
For calibrated cameras:

Cannot transfer all points into the first image.
For calibrated cameras:

Project onto a cylinder instead.
Panoramas

For calibrated cameras:

Distances are roughly preserved. Lines may not appear straight.
To do

- Work on assignment 2